



WORKSHEET

Probability Question – Final Cases

Question Description: This is collection of the other ways they can ask you on probability. By doing this worksheet after the “Probability Distribution Table Worksheet”, “Probability Tree Diagram Worksheet” and “Binomial Distribution Worksheet” you will understand fully how they ask on Probabilities in this S1 exam. The following final probability questions really touch on this part of the syllabus.

Candidates should be able to:

- evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations

Notes and examples

- e.g. the total score when two fair dice are thrown.
- e.g. drawing balls at random from a bag containing balls of different colours.

QUESTIONS

Q1 s2015_p62

A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

- (i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S .

- (ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S . [5]

Q2 w2018_p63

Out of a class of 8 boys and 4 girls, a group of 7 people is chosen at random.

- (i) Find the probability that the group of 7 includes one particular boy. [3]
- (ii) Find the probability that the group of 7 includes at least 2 girls. [4]

Q3 w2015_p63

In country X , 25% of people have fair hair. In country Y , 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y . A person is chosen at random from these 28 million people.

- (i) Find the probability that the person chosen is from country X . [1]
- (ii) Find the probability that the person chosen has fair hair. [2]
- (iii) Find the probability that the person chosen is from country X , given that the person has fair hair. [2]

Q4 w2015_p62

One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

- (i) he gets a green robot on opening his first packet, [1]
- (ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

- (iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

Q5 w2016_p61

Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T ' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

- (i) Find the probability that, when Deeti carries out operation T , she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T .

- (ii) Find $P(X = 1)$. [3]
- (iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

Q6 w2017_p62

A fair tetrahedral die has faces numbered 1, 2, 3, 4. A coin is biased so that the probability of showing a head when thrown is $\frac{1}{3}$. The die is thrown once and the number n that it lands on is noted. The biased coin is then thrown n times. So, for example, if the die lands on 3, the coin is thrown 3 times.

- (i) Find the probability that the die lands on 4 and the number of times the coin shows heads is 2. [3]
- (ii) Find the probability that the die lands on 3 and the number of times the coin shows heads is 3. [1]
- (iii) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. [3]

Q7 s2018_p61

Vehicles approaching a certain road junction from town A can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town A , 55% turn left, 15% turn right and 30% go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.

- (i) Find the probability that, of the next three vehicles approaching the junction from town A , one goes straight on and the other two either both turn left or both turn right. [4]
- (ii) Three vehicles approach the junction from town A . Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on. [4]

Q8 s2018_p62

In a group of students, $\frac{3}{4}$ are male. The proportion of male students who like their curry hot is $\frac{3}{5}$ and the proportion of female students who like their curry hot is $\frac{4}{5}$. One student is chosen at random.

- (i) Find the probability that the student chosen is either female, or likes their curry hot, or is both female and likes their curry hot. [4]

Q9 s2019_p61

Jameel has 5 plums and 3 apricots in a box. Rosa has x plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in x and hence find x . [3]

Q10 w2015_p62

A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

Q11 s2017_p62

Two identical biased triangular spinners with sides marked 1, 2 and 3 are spun. For each spinner, the probabilities of landing on the sides marked 1, 2 and 3 are p , q and r respectively. The score is the sum of the numbers on the sides on which the spinners land. You are given that $P(\text{score is } 6) = \frac{1}{36}$ and $P(\text{score is } 5) = \frac{1}{9}$. Find the values of p , q and r . [6]

Q12 s2017_p63

A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

ANSWERS

Q1

(i)	$P(2\text{Es } 10) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times {}^3C_2 = \frac{3}{5} \quad (0.6)$ OR $P(2\text{Es } 10) = \frac{{}^3C_2 \times {}^2C_1}{{}^5C_3} = \frac{6}{10}$ $= 0.6$ OR 241, 247, 261, 267, 461, 467 = 6 options 124 126 127 146 147 167 246 247 267 467 Prob = 6/10	M1 M1 A1 3 M1 M1 A1 M1 M1 A1	5×4×3 seen in denom Mult a prob by 3C_2 oe Correct answer 3C_x or yC_2 or 2C_1 oe seen mult by $k \geq 1$ in num 5C_3 seen in denom Correct answer List at least 3 of 241, 247, 261, 267, 461, 467 5C_3 or list to get all 10 options in denom see below Correct answer								
(ii)	124 126 127 146 147 167 246 247 267 467 <table border="1" data-bbox="300 1859 762 1926"> <tr> <td>s</td><td>1</td><td>2</td><td>4</td></tr> <tr> <td>$P(S=s)$</td><td>6/10</td><td>3/10</td><td>1/10</td></tr> </table>	s	1	2	4	$P(S=s)$	6/10	3/10	1/10	M1 A1 B1 B1 B1 5	Attempt at listing with at least 7 correct All correct and no others or all 60 1, 2, 4 only seen in top row Any two correct All correct
s	1	2	4								
$P(S=s)$	6/10	3/10	1/10								

Q2

(i)	Total number of selections = ${}^{12}C_7 = 792$	B1	Seen as denominator of fraction
	Selections with boy included = ${}^{11}C_6$ or ${}^{12}C_7 - {}^{11}C_7 = 462$	M1	Correct unsimplified expression for selections with boy included seen as numerator of fraction
	Probability = $462/792 = 7/12$ (0.583)	A1	Correct answer
	OR		
	prob of boy not included = $11/12 \times 10/11 \times \dots \times 5/6 = 5/12$	B1	Correct unsimplified prob
	$1 - 5/12$	M1	Subtracting prob from 1
	$= 7/12$	A1	Correct answer
		3	
(ii)	Method 1		
	Scenarios are: 2G + 5B: ${}^4C_2 \times {}^8C_5 = 336$	B1	One unsimplified product correct
	3G + 4B: ${}^4C_3 \times {}^8C_4 = 280$ 4G + 3B: ${}^4C_4 \times {}^8C_3 = 56$	M1	No of selections (products of nC_r and nP_r) added for 2, 3 and 4 girls with no of girls and no of boys summing to 7
	Total = 672	A1	Correct total
	Probability = $672/792$ (28/33) (0.848)	A1ft	Correct answer – ‘total’/ (‘total no of selections’ from i)
	Method 2		
	0G + 7B ${}^4C_0 \times {}^8C_7 = 8$	B1	One unsimplified no of selections correct
	1G + 6B ${}^4C_1 \times {}^8C_6 = 112$ Total = $8 + 112 = 120$	M1	No of selections (products of nC_r and nP_r) added for 0 and 1 girls with no of girls and no of boys summing to 7
	$({}^{12}C_7 - 120)/792$ or $1 - 120/792$	A1	$792 - 120 = 672$ or $1 - 120/792$
	Probability = $672/792$ (28/33) (0.848)	A1ft	‘672’ over ‘792’ from i
	Method 3 (probability)		
	$1 - P(0) - P(1)$ $= 1 - (8/12 \times 7/11 \times \dots \times 2/6) - (8/12 \times \dots \times 3/7 \times 4/6 \times 7)$	B1	One correct unsimplified prob for 0 or 1
	$= 1 - 1/99 - 14/99$	M1	Subtracting ‘P(0)’ and ‘P(1)’ (using products of 7 fractions with denominators from 12 to 6) from 1
		A1	Both probs correct unsimplified
	$= 84/99 = 28/33$	A1ft	$1 - ‘P(0)’ - ‘P(1)’$

Q3

(i)	$P(X) = \frac{20}{28} \left(\frac{5}{7} \right) (0.714), 71.4\%$	B1	1	oe
(ii)	$P(F) = \frac{20}{28} \times \frac{1}{4} \times \frac{8}{28} \times \frac{6}{10} = \frac{7}{20}$	M1		Summing two 2-factor probs created by One of $\frac{1}{4}$ or $\frac{3}{4}$ multiplied by $\frac{20}{28}$ or $\frac{8}{28}$ Added to $\frac{4}{10}$ or $\frac{6}{10} \times$ altn population prob
		A1	2	Correct answer
(iii)	$P(X F) = \frac{5/28}{7/20} = \frac{25}{49} (0.510)$	M1		Their unsimplified country X probability ($\frac{5}{28}$) as num or denom of a fraction Or (their fair hair population) $\div$ (total fair hair pop)
		A1	2	Correct answer

Q4

(i)	$\frac{1}{4}$	B1	1	
(ii)	$\left(\frac{3}{4} \right)^4 \left(\frac{1}{4} \right) = \frac{81}{1024} = 0.0791$	M1		Expression of form $p^4(1-p)$ only, $p = 1/4$ or $3/4$
		A1	2	Correct answer
(iii)	$P(\text{all diff}) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times 4!$ $= \frac{3}{32} (0.0938)$	M1		4! on numerator seen mult by $k \geq 1$ or $3 \times 2 \times 1$ on num oe, must be in a fraction.
	OR $1 \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{3}{32}$	M1		4^4 on denom or 4^3 on denom with the $3 \times 2 \times 1$
		A1	3	Correct answer

Q5

(i)	$P(B, B) = 1/4 \times 2/5$	M1		Multiplying two different probs
	$= 1/10$	A1	[2]	
(ii)	$P(X=1) = P(R,R) + P(B,B)$	M1		Finding $P(R, R)$ ($=3/5$)
	$= 3/4 \times 4/5 + 1/10$ $= 14/20$ (7/10)	M1 A1	[3]	Summing two options
(iii)	$P(B \cap B)$	M1		their (i) seen as num or denom of a fraction
	$= \frac{P(B \cap B)}{P(B)} = \frac{1/10}{3/4 \times 1/5 + 1/4 \times 2/5}$	M1		$3/4 \times p_1 + 1/4 \times p_2$ seen anywhere
		A1		1/4 (unsimplified) seen as num or denom of a fraction, www
	$= 2/5$	A1	[4]	

Q6

(i)	$P(4, 2H) = \frac{1}{4} \times {}^4C_2 \times \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$	M1	Multiplying their 2H expression by $1/4$ [P(4)]
		M1	Remaining factor is $\left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$ [or $\frac{4}{81}$] multiplied by integer value $k \geq 1$ OE
	$= \frac{2}{27}$ (0.0741)	A1	
(ii)		3	
	$P(3, 3H) = \frac{1}{4} \times \left(\frac{1}{3}\right)^3 = \frac{1}{108}$ (0.00926)	B1	
(iii)		1	
	$P(1, 1H) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (0.08333)	M1	Correct expression for 1 of $P(1, 1H)$, $P(2, 2H)$, $P(4, 4H)$ Unsimplified (or better)
	$P(2, 2H) = \frac{1}{4} \times \left(\frac{1}{3}\right)^2 = \frac{1}{36}$ (0.02778)	M1	Summing their values for 3 or 4 appropriate outcomes for the 'game' with no additional outcomes.
	$P(3, 3H) = \frac{1}{4} \times \left(\frac{1}{3}\right)^3 = \frac{1}{108}$ (0.009259)		
	$P(4, 4H) = \frac{1}{4} \times \left(\frac{1}{3}\right)^4 = \frac{1}{324}$ (0.003086)		
	$\text{Prob} = \frac{10}{81}$ (0.123)	A1	
		3	

Q7

(i)	$P(\text{SLL}) = (0.3)(0.55)(0.55) = 0.09075 \left(\frac{363}{4000} \right)$	M1	P(SLL), P(SRR), P(SSL) or P(SSR) seen
	$P(\text{SRR}) = (0.3)(0.15)(0.15) = 0.00675 \left(\frac{27}{4000} \right)$	A1	Two correct options 0.09075 or 0.00675 can be unsimplified
	Total = ${}^3C_1 \times P(\text{SLL}) + {}^3C_1 \times P(\text{SRR})$ = $0.27225 + 0.02025$	M1	Summing 6 prob options not all identical
	Prob = 0.293 accept 0.2925 $\left(\frac{117}{400} \right)$	A1	Correct answer
		4	
(ii)	$P(\text{SSS} \mid \text{all same dir}^n) = \frac{P(\text{SSS and same dir}^n)}{P(\text{same direction})}$	B1	$(0.3)^3$ oe seen on its own as num or denom of a fraction
		M1	Attempt at $P(\text{SSS} + \text{LLL} + \text{RRR})$ seen anywhere
	$= \frac{0.3 \times 0.3 \times 0.3}{(0.15)^3 + (0.55)^3 + (0.3)^3}$	A1	$(0.15)^3 + (0.55)^3 + (0.3)^3$ oe seen as denom of a fraction
	$= 0.137 \left(\frac{108}{787} \right)$	A1	Correct answer
		4	

Q8

(i)	Method 1 $P(M \cap H) = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20} \quad (0.45)$	B1	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = \frac{1}{4} + \frac{9}{20} = \frac{14}{20}$	M1	Numerical attempt at $P(F) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10} \quad (0.7) \text{ OE}$	A1	Correct final answer
	Method 2 $P(M \cap H') = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20} \quad (0.3)$	B1	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = 1 - P(M \cap H')$	M1	Numerical attempt at $1 - P(M \cap H')$
	$= 1 - \frac{3}{4} \times \frac{2}{5}$	A1	Correct unsimplified expression
	$= \frac{7}{10} \quad (0.7) \text{ OE}$	A1	Correct final answer

Q9

Jameel: $P(\text{plum}) = \frac{5}{8}$, Rosa: $P(\text{plum}) = \frac{x}{x+6}$	M1	<i>Their 2 probabilities for P(plum) multiplied and equated to 1/4</i>
$\frac{5}{8} \times \frac{x}{x+6} = \frac{1}{4}$	A1	Correct equation oe
$(x =) 4$	A1	SC correct answer with no appropriate equations i.e. common sense B1
	3	

Q10

$P(\text{no men}) \frac{{}^9C_6}{{}^{16}C_6} = \frac{84}{8008} = \frac{21}{2002} = \frac{3}{286}$ $= 0.0105$	B1	9C_6 seen anywhere
	B1	${}^{16}C_6$ seen as denom of fraction oe
	B1	3 Correct final answer
OR $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \times \frac{5}{12} \times \frac{4}{11} = 0.0105$	B1	
	B1	$(9 \times 8 \times 7 \times 6 \times 5 \times 4)$ seen anywhere
	B1	Correct unsimplified denom
		Correct final answer

Q11

$P(\text{score is } 6) = P(3, 3)$	M1	Realising that score 6 is only $P(3, 3)$
$= r^2 = 1/36$ $r = 1/6$	A1	Correct ans [SR B2 $r = 1/6$ without workings]
$P(2, 3) + P(3, 2) = 1/9$ $qr + rq = 1/9$	M1	Eqn involving qr (OE) equated to $1/9$ (r may be replaced by <i>their</i> 'r value')
$q/6 + q/6 = 1/9$	M1	Correct equation with <i>their</i> 'r value' substituted
$q = 1/3$	A1	Correct answer seen, does not imply previous M's
$p = 1 - 1/6 - 1/3 = 1/2$	B1 FT	FT their p + their r + their $q = 1$, $0 < p < 1$
Total:	6	

Q12

$P(6) = 0.3$	B1	SOI
$P(\text{sum is } 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3)$	M1	Identifying the four ways of summing to 9 (3,6), (6,3) (4,5) and (5,4)
$= (0.03 + 0.02) \times 2$	M1	Mult 2 probs together to find one correct prob of (3,6), (6,3) (4,5) or (5,4) unsimplified
$= 0.1$	A1	OE
Total:	4	